

# The total energy-momentum of the universe in teleparallel gravity

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## Abstract

We investigate the conservation law of energy-momentum in teleparallel gravity by using general Noether theorem. The energy-momentum current has also superpotential and is therefore identically conserved. The total energy-momentum, which includes the contributions of both matter and gravitational fields, is given by the integral of scalar densities over a three-dimensional spacelike hypersurface. As an example, the universe in teleparallel gravity is investigated. It is shown that the total energy-momentum vanishes independently of both the curvature parameter and the three dimensionless coupling constants of teleparallel gravity.

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## I. INTRODUCTION

The definition of energy-momentum density for the gravitational field is one of the most fundamental and controversial problems in general relativity. However, some attempts to the problem leads to prescriptions that are not true tensors. The first of such attempts was made by Einstein who proposed an expression for the energy-momentum distribution of the gravitational field. There have been many attempts to resolve the energy problem in Einstein's theory of General Relativity.

Teleparallel theories of gravity, which based on Weitzenböck geometry [1], have been considered long time ago in connection with attempts to define the energy of the gravitational field [2]. In the theory of the teleparallel gravity, the curvature tensor vanishes identically and gravitation is attributed to torsion [3]. Furthermore, the fundamental field in this theory is a nontrivial tetrad rather than the metric. It is known that there exists no covariant, nontrivial expression constructed out of the metric tensor. However, covariant expressions that contain second order derivatives of tetrad fields are feasible [4].

The teleparallel equivalent of general relativity (TEGR) [5, 6, 7, 8, 9, 10, 11, 12] is an alternative geometrical description of Einstein's theory. Recently, a method for dealing with the localization of the gravitational energy had been presented in the Lagrangian framework of the TEGR by Andrade, Guillen and Pereira [13]. They obtained an energy-momentum gauge current for the gravitational field. The expression is a true space-time and gauge tensor, can be reduced to Møller's energy-momentum density of the gravitational field.

Subsequently, Blagojević and Vasilic investigated the conservation laws associated with the asymptotic Poincaré symmetry of spacetime in the general teleparallel theory of gravity [14]. They obtained the improved form of the canonical Poincaré generators, which defines the conserved charges of the theory. While Maluf and da Rocha-Neto considered the Hamiltonian formulation of the teleparallel equivalent of general relativity [15]. The gravitational energy-momentum is given by the integral of scalar densities over a three-dimensional spacelike hypersurface.

In this paper, we would like to re-examine the energy-momentum problem of teleparallel gravity with general Noether theorem. Our purpose is to present the relationship between conservation theorems and invariance properties of physical systems in teleparallel theory. We will prove that the energy-momentum current has also superpotential and is therefore

identically conserved.

The paper is arranged as follows. In Sec. II, a brief review of teleparallel gravity is given. In Sec. III, we give a general description of the scheme for establishing covariant conservation laws in gravitational theory. In Sec. IV, we use the scheme to obtain a conservation law of energy-momentum in teleparallel gravity. In Sec. V, we calculate the total energy and momentum of the universe in teleparallel gravity by superpotential. Sec. VI is devoted to some remarks and discussions.

## II. REVIEW OF TELEPARALLEL GRAVITY

Let us start by giving a simple review of the teleparallel gravity theory (for the details, see Ref. [29]). We use the Greek alphabet ( $\mu, \nu, \lambda, \dots = 0, 1, 2, 3$ ) to denote indices related to spacetime, and the Latin alphabet ( $a, b, c, \dots = 0, 1, 2, 3$ ) to denote indices related to the tangent space, assumed to be a Minkowski space with the metric  $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$ . In the theory of the teleparallel gravity, spacetime is represented by the Weitzenböck manifold  $W^4$ , and the action is given by

$$S = \int d^4x h \left( \frac{1}{16\pi G} S^{\lambda\mu\nu} T_{\lambda\mu\nu} + \mathcal{L}_M \right), \quad (1)$$

where  $h = \det(h^a_\mu)$  with  $h^a_\mu$  a tetrad field which satisfies  $g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu$ ,  $\mathcal{L}_M$  is the Lagrangian of the matter field, and  $S^{\lambda\mu\nu}$  is the tensor defined by the torsion  $T^{\lambda\mu\nu}$  of the Weitzenböck connection  $\Gamma^\lambda_{\mu\nu}$

$$S^{\lambda\mu\nu} = c_1 T^{\lambda\mu\nu} + \frac{c_2}{2} (T^{\mu\lambda\nu} - T^{\nu\lambda\mu}), \\ + \frac{c_3}{2} (g^{\lambda\nu} T^{\sigma\mu}_\sigma - g^{\lambda\mu} T^{\sigma\nu}_\sigma) \quad (2)$$

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu}, \quad (3)$$

$$\Gamma^\lambda_{\mu\nu} = h_a{}^\lambda \partial_\nu h^a_\mu, \quad (4)$$

with  $c_1$ ,  $c_2$ , and  $c_3$  three dimensionless coupling constants [3]. It is well known that the Weitzenböck connection presents torsion but no curvature [16] and the curvature of the Weitzenböck connection vanishes identically as a consequence of absolute parallelism. It is important to remark that, in this theory, the fundamental field is a nontrivial tetrad rather than the metric. For the specific choice

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1, \quad (5)$$

teleparallel gravity reduces to the so called teleparallel equivalent of general relativity.

### III. CONSERVATION LAWS IN GRAVITATIONAL THEORY

The conservation law is one of the important problems in gravitational theory. It is due to the invariance of the action corresponding to some transforms. In order to study the covariant energy-momentum law of special systems, it is necessary to discuss conservation laws by Noether theorem in the general case [17, 18, 19, 20, 21]. The action of a system is

$$I = \int_{\mathcal{M}} d^4x \mathcal{L}(\phi^A, \partial_\mu \phi^A), \quad (6)$$

where  $\phi^A$  are independent variables with general index  $A$  and denote the general fields. If the action is invariant under the infinitesimal transformations

$$x'^\mu = x^\mu + \delta x^\mu, \quad (7)$$

$$\phi'^A(x') = \phi^A(x) + \delta \phi^A(x), \quad (8)$$

and  $\delta \phi^A$  vanishes on the boundary of  $\mathcal{M}$ ,  $\partial \mathcal{M}$ , then following relation holds [17, 19, 22, 23, 24]

$$\partial_\mu (\mathcal{L} \delta x^\mu + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^A} \delta_0 \phi^A) + [\mathcal{L}]_{\phi^A} \delta_0 \phi^A = 0, \quad (9)$$

where

$$[\mathcal{L}]_{\phi^A} = \frac{\partial \mathcal{L}}{\partial \phi^A} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^A}, \quad (10)$$

and  $\delta_0 \phi^A$  is the Lie variative of  $\phi^A$

$$\delta_0 \phi^A = \phi'^A(x) - \phi^A(x) = \delta \phi^A(x) - \partial_\mu \phi^A \delta x^\mu. \quad (11)$$

If  $\mathcal{L}$  is the total Lagrangian density of the system, the field equation of  $\phi^A$  is just  $[\mathcal{L}]_{\phi^A} = 0$ . Hence from Eq. (9), we can obtain the conservation equation corresponding to transformations (7) and (8)

$$\partial_\mu (\mathcal{L} \delta x^\mu + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^A} \delta_0 \phi^A) = 0. \quad (12)$$

It is important to recognize that if  $\mathcal{L}$  is not the total Lagrangian density, such as the gravitational part  $\mathcal{L}_g$ , then so long as the action of  $\mathcal{L}_g$  remains invariant under transformations (7) and (8), Eq. (9) is still valid yet Eq. (12) is no longer admissible because of  $[\mathcal{L}_g]_{\phi^A} \neq 0$ .

In gravitational theory with the tetrad as elementary fields, we can separate  $\phi^A$  as  $\phi^A = (h_a^\mu, \psi^B)$ , where  $\psi^B$  is an arbitrary tensor under general coordinate transformations. Suppose that  $\mathcal{L}_g$  does not contain  $\psi^B$ , then Eq. (9) reads

$$\partial_\mu \left( \mathcal{L}_g \delta x^\mu + \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} \delta_0 h_a^\nu \right) + [\mathcal{L}_g]_{h_a^\nu} \delta_0 h_a^\nu = 0. \quad (13)$$

Under transformations (7) and (8), the Lie variations are

$$\delta_0 h_a^\mu = h_a^\nu \partial_\nu \delta x^\mu - \delta x^\nu \partial_\nu h_a^\mu, \quad (14)$$

Substituting Eq. (14) into Eq. (13) gives

$$\begin{aligned} \partial_\mu \left[ \left( \mathcal{L}_g \delta_\sigma^\mu - \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} \partial_\sigma h_a^\nu \right) \delta x^\sigma + \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} h_a^\sigma \partial_\sigma \delta x^\nu \right] \\ + [\mathcal{L}_g]_{h_a^\mu} (h_a^\nu \partial_\nu \delta x^\mu - \delta x^\nu \partial_\nu h_a^\mu) = 0. \end{aligned} \quad (15)$$

Comparing the coefficients of  $\delta x^\mu$ ,  $\delta x_{,\nu}^\mu$  and  $\delta x_{,\nu\lambda}^\mu$ , we can obtain an identity

$$[\mathcal{L}_g]_{h_a^\nu} \partial_\mu h_a^\nu + \partial_\nu ([\mathcal{L}_g]_{h_a^\mu} h_a^\nu) = 0. \quad (16)$$

Then Eq. (15) can be written as

$$\begin{aligned} \partial_\mu \left[ \left( \mathcal{L}_g \delta_\sigma^\mu - \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} \partial_\sigma h_a^\nu + [\mathcal{L}_g]_{h_a^\sigma} h_a^\mu \right) \delta x^\sigma \right. \\ \left. + \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} h_a^\sigma \partial_\sigma \delta x^\nu \right] = 0. \end{aligned} \quad (17)$$

This is the general conservation law in the tetrad formalism of spacetime. By definition, we introduce

$$\tilde{I}_\sigma^\mu \equiv \mathcal{L}_g \delta_\sigma^\mu - \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} \partial_\sigma h_a^\nu + [\mathcal{L}_g]_{h_a^\sigma} h_a^\mu, \quad (18)$$

$$\tilde{V}_\nu^{\mu\sigma} \equiv \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_a^\nu} h_a^\sigma. \quad (19)$$

Then Eq. (17) gives

$$\partial_\mu (\tilde{I}_\sigma^\mu \delta x^\sigma + \tilde{V}_\nu^{\mu\sigma} \partial_\sigma \delta x^\nu) = 0. \quad (20)$$

Eq. (20) is tenable under arbitrary infinitesimal transformations, so we can compare the coefficients of  $\delta x^\sigma$ ,  $\delta x_{,\mu}^\sigma$  and  $\delta x_{,\mu\lambda}^\sigma$  and obtain

$$\partial_\mu \tilde{I}_\sigma^\mu = 0, \quad (21)$$

$$\tilde{I}_\sigma^\mu = -\partial_\nu \tilde{V}_\sigma^{\nu\mu}, \quad (22)$$

$$\tilde{V}_\nu^{\mu\sigma} = -\tilde{V}_\nu^{\sigma\mu}. \quad (23)$$

Eqs. (21)-(23) are fundamental to the establishing of conservation law in the theory of gravitation.

#### IV. CONSERVATION LAW OF ENERGY-MOMENTUM IN TELEPARALLEL GRAVITY

From Eqs. (1) and (2), we can rewrite the gauge gravitational field Lagrangian as follows:

$$\mathcal{L}_g = \frac{h}{16\pi G} (c_1 T^{\lambda\mu\nu} T_{\lambda\mu\nu} + c_2 T^{\mu\lambda\nu} T_{\lambda\mu\nu} + c_3 T^{\nu\mu}{}_{\nu} T^{\lambda}{}_{\mu\lambda}). \quad (24)$$

The further expression of  $\mathcal{L}_g$  is

$$\mathcal{L}_g = \frac{h}{16\pi G} (c_1 T^{abc} T_{abc} + c_2 T^{bac} T_{abc} + c_3 T^{ab}{}_a T^c{}_{bc}), \quad (25)$$

with  $T_{abc}$  defined as

$$T_{abc} = h_{a\mu} (h_c{}^\nu \partial_\nu h_b{}^\mu - h_b{}^\nu \partial_\nu h_c{}^\mu). \quad (26)$$

For transformations  $x'^\mu = x^\mu + h_a{}^\mu b^a$ , Eq. (20) implies

$$\partial_\mu (\tilde{I}_\sigma^\mu h_a{}^\sigma + \tilde{V}_\sigma^{\mu\nu} \partial_\nu h_a{}^\sigma) = 0. \quad (27)$$

From Einstein equations  $hT_a^\mu = [\mathcal{L}_g]_{h^a{}_\mu}$  and Eq. (18), we can express  $\tilde{I}_\nu^\mu h_a{}^\nu$  as

$$\tilde{I}_\nu^\mu h_a{}^\nu = \left( \mathcal{L}_g \delta_\nu^\mu - \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h^a{}_\lambda} \partial_\nu h^a{}_\lambda \right) h_a{}^\nu + hT_a^\mu. \quad (28)$$

Defining

$$h t_a^\mu = \left( \mathcal{L}_g \delta_\nu^\mu - \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h^a{}_\lambda} \partial_\nu h^a{}_\lambda \right) h_a{}^\nu + \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_b{}^\nu} h_b{}^\sigma \partial_\sigma h_a{}^\nu, \quad (29)$$

and considering Eq. (19), we then have

$$\tilde{I}_\sigma^\mu h_a{}^\sigma + \tilde{V}_\sigma^{\mu\nu} \partial_\nu h_a{}^\sigma = h(T_a^\mu + t_a^\mu). \quad (30)$$

So Eq. (27) can be written as

$$\partial_\mu [h(T_a^\mu + t_a^\mu)] = 0, \quad (31)$$

or

$$\nabla_\mu (T_a^\mu + t_a^\mu) = 0. \quad (32)$$

This equation is the desired covariant conservation law of energy-momentum in a teleparallel gravity system.  $t_a^\mu$  defined in Eq. (29) is the energy-momentum density of gravity field, and

$T_a^\mu$  to that of matter part. By virtue of Eq. (22), the expression on the LHS of Eq. (30) can be expressed as divergence of superpotential  $V_a^{\mu\nu}$

$$h(T_a^\mu + t_a^\mu) = \partial_\nu V_a^{\mu\nu}, \quad (33)$$

where

$$V_a^{\mu\nu} = \tilde{V}_\sigma^{\mu\nu} h_a^\sigma = \frac{\partial \mathcal{L}_g}{\partial \partial_\mu h_b^\sigma} h_b^\nu h_a^\sigma. \quad (34)$$

Eq. (33) shows that the total energy-momentum density of a gravity system always can be expressed as divergence of superpotential. The total energy-momentum is

$$P_a = \int_\Sigma d\Sigma_\mu h(T_a^\mu + t_a^\mu) = \int_S dS_{\mu\nu} V_a^{\mu\nu}, \quad (35)$$

where  $dS_{\mu\nu} = \frac{1}{3!} \varepsilon_{\mu\nu\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma$ . This conservation law of energy-momentum in general relativity has the following main properties: It is a covariant definition with respect to general coordinate transformations. But the energy-momentum tensor is not covariant under local Lorentz transformations, this is reasonable because of the equivalence principle.

Now we calculate the expressions of  $V_a^{\mu\nu}$  by using the gravity Lagrangian density (25) of teleparallel gravity. The explicit expressions are

$$\begin{aligned} V_a^{\mu\nu} = \frac{h}{8\pi G} & \left[ (h_c^\mu h_b^\nu - h_c^\nu h_b^\mu) (c_1 T_a^{bc} + c_2 T^b{}_a{}^c) \right. \\ & \left. + c_3 (h_a^\mu h_b^\nu - h_a^\nu h_b^\mu) T^{cb}{}_c \right]. \end{aligned} \quad (36)$$

## V. THE ENERGY-MOMENTUM OF THE UNIVERSE IN TELEPARALLEL GRAVITY

About two decades ago, Rosen [25] considered a closed homogeneous isotropic universe described by the Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = dt^2 - \frac{a(t)^2}{(1 + r^2/4)^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2). \quad (37)$$

Then using Einstein's prescription, he obtained the following energy-momentum complex

$$\Theta_0^0 = \frac{a}{8\pi} \left[ \frac{3}{(1 + r^2/4)^2} - \frac{r^2}{(1 + r^2/4)^3} \right]. \quad (38)$$

By integrating the above over all space, one finds that the total energy  $E$  of the universe is zero. These interesting results fascinated some general relativists, for instance, Johri

*et al.* [26], Banerjee and Sen [27] and Xulu [28]. Johri *et al.* [26], using the Landau and Lifshitz energy-momentum complex, showed that the total energy of an FRW spatially closed universe is zero at all times irrespective of equations of state of the cosmic fluid. They also showed that the total energy enclosed within any finite volume of the spatially flat FRW universe is zero at all times.

Recently, Vargas [29] considered the teleparallel version of both Einstein and Landau-Lifshitz energy-momentum complexes. His basic result is that the total energy vanishes whatever be the pseudotensor used to describe the gravitational energy. It is also independent of both the curvature parameter and the three teleparallel dimensionless coupling constants. But he worked with Cartesian coordinates, as other coordinates may lead to non-physical values for pseudotensor, As remarked in Ref. [30].

In this section we calculate the total energy-momentum of the homogeneous isotropic FRW universe by our conservation law in two kinds of coordinates: sphere coordinates and Cartesian coordinates.

### A. The energy-momentum in sphere coordinates

The line element of the homogeneous isotropic FRW universe is given by

$$ds^2 = dt^2 - \frac{a(t)^2}{(1 + kr^2/4)^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (39)$$

where  $a(t)$  is the time-dependent cosmological scale factor, and  $k$  is the curvature parameter  $k = 0, \pm 1$ . The tetrad components are

$$h^a{}_\mu = \text{diag} \left( 1, \frac{a(t)}{1 + kr^2/4}, \frac{ra(t)}{1 + kr^2/4}, \frac{ra(t) \sin \theta}{1 + kr^2/4} \right). \quad (40)$$

Their inverses are

$$h_a{}^\mu = \text{diag} \left( 1, \frac{1 + kr^2/4}{a(t)}, \frac{1 + kr^2/4}{ra(t)}, \frac{1 + kr^2/4}{ra(t) \sin \theta} \right). \quad (41)$$

From Eqs. (40) and (41), we can now construct the Weitzenböck torsion  $T_{abc}$ , whose nonvanishing components are

$$\begin{aligned} T_{101} &= T_{202} = T_{303} = -\frac{\dot{a}(t)}{a(t)}, \\ T_{212} &= T_{313} = \frac{-4 + kr^2}{4ra(t)}, \\ T_{323} &= -\frac{4 + kr^2}{4ra(t)} \cot \theta, \end{aligned} \quad (42)$$



where a dot denotes a derivative with respect to the time  $t$ . For  $T_a = T_{ab}^b$ , the calculated result is as follows

$$T_0 = 3\frac{\dot{a}(t)}{a(t)}, \quad T_1 = \frac{4 - kr^2}{2ra(t)}, \quad T_2 = \frac{4 + kr^2}{4ra(t)} \cot \theta. \quad (43)$$

For superpotential  $V_a^{\mu\nu}$ , its non-zero components are

$$\begin{aligned} V_0^{01} &= \frac{c_3(-4 + kr^2)ra(t) \sin \theta}{\pi G(4 + kr^2)^2}, \quad V_2^{02} = V_1^{01}/r, \\ V_0^{02} &= -\frac{c_3 a(t) \cos \theta}{2\pi G(4 + kr^2)}, \quad V_1^{12} = -\frac{c_3 \cos \theta}{8\pi G}, \\ V_1^{01} &= -\frac{2(2c_1 + c_2 + 3c_3)r^2 \dot{a}(t) \sin \theta}{\pi G(4 + kr^2)^2}, \\ V_2^{12} &= -\frac{(2c_1 + c_2 + 3c_3)(-4 + kr^2) \sin \theta}{8\pi G(4 + kr^2)}, \\ V_3^{03} &= V_1^{01}/(r \sin \theta), \quad V_3^{13} = V_2^{12}/\sin \theta. \\ V_3^{23} &= -\frac{(2c_1 + c_2 + 3c_3) \cot \theta}{8\pi Gr}, \end{aligned} \quad (44)$$

Let us now calculate the total energy-momentum of the FRW universe at the instant  $x^0 = t = \text{constant}$ , which is given by the integral over the space section or at the infinite of the space. At the region of the integral, i.e.  $S = \partial\Sigma = \Sigma|_{r \rightarrow \infty}$ , we have  $dt = dr = 0$ , and Eq. (35) becomes

$$P_a = \int_S dS_{01} V_a^{01} = \int_S d\theta d\varphi V_a^{01} = \lim_{r \rightarrow \infty} \int_0^\pi d\theta \int_0^{2\pi} d\varphi V_a^{01}. \quad (45)$$

Substituting above calculated results of  $V_a^{01}$  into Eq. (45) yields

$$P_a = (0, 0, 0, 0). \quad (46)$$

So, the total energy and momentum of the closed ( $k=1$ ), open ( $k = -1$ ) and spatially flat ( $k = 0$ ) universes vanish.

## B. The energy-momentum in Cartesian coordinates

Transforming from polar to Cartesian coordinates, the FRW line element (39) becomes

$$ds^2 = dt^2 - \frac{a(t)^2}{(1 + kr^2/4)^2} (dx^2 + dy^2 + dz^2). \quad (47)$$

The tetrad components and their inverses are

$$h^a{}_\mu = \text{diag} \left( 1, \frac{a(t)}{1 + kr^2/4}, \frac{a(t)}{1 + kr^2/4}, \frac{a(t)}{1 + kr^2/4} \right), \quad (48)$$

$$h_a{}^\mu = \text{diag} \left( 1, \frac{1 + kr^2/4}{a(t)}, \frac{1 + kr^2/4}{a(t)}, \frac{1 + kr^2/4}{a(t)} \right). \quad (49)$$

From these above two equations, the nonvanishing components of  $T_{abc}$  and  $T_a$  are constructed as follows:

$$\begin{aligned} T_{101} &= T_{202} = T_{303} = -\frac{\dot{a}(t)}{a(t)}, \quad T_{221} = T_{331} = -\frac{kx}{2a(t)}, \\ T_{112} &= T_{332} = -\frac{ky}{2a(t)}, \quad T_{113} = T_{223} = -\frac{kz}{2a(t)}, \\ T_0 &= 3\frac{\dot{a}(t)}{a(t)}, \quad T_a = -\frac{kx^i \delta_{ia}}{a(t)}, \quad (a, i = 1, 2, 3) \end{aligned} \quad (50)$$

For superpotential  $V_a^{\mu\nu}$ , its non-zero components are

$$\begin{aligned} V_0^{0i} &= \frac{2c_3 k a x^i}{\pi G (4 + kr^2)^2}, \quad (i = 1, 2, 3) \\ V_a^{0i} &= -\frac{2(2c_1 + c_2 + 3c_3) \dot{a} a}{\pi G (4 + kr^2)^2} \delta_a^i, \quad (i, a = 1, 2, 3) \\ V_2^{21} &= V_3^{31} = \frac{(2c_1 + c_2 + 2c_3) k x}{4\pi G (4 + kr^2)}, \\ V_1^{12} &= V_3^{32} = \frac{(2c_1 + c_2 + 2c_3) k y}{4\pi G (4 + kr^2)}, \\ V_1^{13} &= V_2^{23} = \frac{(2c_1 + c_2 + 2c_3) k z}{4\pi G (4 + kr^2)}. \end{aligned} \quad (51)$$

Now we can calculate the total energy-momentum. In Cartesian coordinates,  $dS_{0i} = x^i r d\Omega = x^i r \sin \theta d\theta d\varphi$ , so Eq. (35) becomes

$$\begin{aligned} P_a &= \int_S dS_{0i} V_a^{0i} = \int_S d\theta d\varphi x^i r \sin \theta V_a^{0i} \\ &= \lim_{r \rightarrow \infty} \int_0^\pi d\theta \int_0^{2\pi} d\varphi (x^i r \sin \theta V_a^{0i}). \end{aligned} \quad (52)$$

Substituting above calculated results into Eq. (52) yields again

$$P_a = (0, 0, 0, 0). \quad (53)$$

So, the values of the total energy and momentum of the FRW universe, which calculated in Cartesian coordinate, vanish too.

## VI. SUMMARY AND DISCUSSIONS

To summarize, by the use of general Noether theorem, we have obtained the conservation law of energy-momentum in teleparallel gravity theory. The energy-momentum current has also superpotential and is therefore identically conserved. Based on this conservation law of energy-momentum, we have calculated the total energy and momentum of the FRW universe, which includes contributions of matter and gravitational field. All calculated values vanish. They are also independent of both the curvature parameter and the three teleparallel dimensionless coupling constants. Therefore it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model. Commonly, the universe is filled with matter field and gravitational field, but its total energy is actually zero. Thus we can conclude that the gravitational energy exactly cancels out the matter energy.

It is important to remark that, all results, calculated in both sphere and Cartesian coordinates, are same. So the corresponding conservative quantities are independent of the choice of coordinates, which are caused by the covariance of the conservation law. We think a covariant conservation law of angular momentum is still needed in order to understand the conservative quantities in the theory of teleparallel gravity.

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